Here is a summary of the **fundamental identities.**

Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}, \quad \sin\theta \neq 0$

$\sec\theta = \dfrac{1}{\cos\theta}, \quad \cos\theta \neq 0$

$\cot\theta = \dfrac{1}{\tan\theta}, \quad \tan\theta \neq 0$

Ratio Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \quad \cos\theta \neq 0$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}, \quad \sin\theta \neq 0$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

Odd-Even Identities

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

CLASS EXERCISES

Complete by writing 1 or −1 to make each sentence an identity.

1. $\sin\theta\csc\theta = \underline{?}$

2. $\sec^2\theta - \tan^2\theta = \underline{?}$

3. $\dfrac{\sin(-\theta)}{\sin\theta} = \underline{?} \quad (\sin\theta \neq 0)$

4. $\cot\theta\tan\theta = \underline{?}$

5. $\tan^2\theta - \sec^2\theta = \underline{?}$

6. $\cos^2\theta + \sin^2\theta = \underline{?}$

7. $\cot^2\theta - \csc^2\theta = \underline{?}$

8. $\sec\theta\cos\theta = \underline{?}$

9. $\dfrac{\cos\theta}{\cos(-\theta)} = \underline{?} \quad (\cos(-\theta) \neq 0)$

10. $\dfrac{\sin\theta}{\cos\theta\tan\theta} = \underline{?} \quad (\cos\theta\tan\theta \neq 0)$

PRACTICE EXERCISES

Write two equivalent ways to express each identity.

1. $\csc\theta = \dfrac{1}{\sin\theta}, \quad \sin\theta \neq 0$

2. $\sec\theta = \dfrac{1}{\cos\theta}, \quad \cos\theta \neq 0$

3. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \quad \cos\theta \neq 0$

4. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}, \quad \sin\theta \neq 0$

5. $\sin^2\theta + \cos^2\theta = 1$

6. $1 + \cot^2\theta = \csc^2\theta$

7. $1 + \tan^2\theta = \sec^2\theta$

8. Use the two ratio identities to show that $(\tan\theta)(\cot\theta) = 1$.

Verify each identity for the given angle measure.

9. $1 + \cot^2 30° = \csc^2 30°$

10. $1 + \tan^2 30° = \sec^2 30°$

11. $\sin^2 45° + \cos^2 45° = 1$

12. $\cot 45° = \dfrac{\cos 45°}{\sin 45°}$

13. $\tan(-60°) = \dfrac{\sin(-60°)}{\cos(-60°)}$

14. $\sec(-60°) = \dfrac{1}{\cos(-60°)}$

15. $\sin(-120°) = -\sin 120°$

16. $\cos(-120°) = \cos 120°$

17. $1 - \sin^2\left(\dfrac{3\pi}{4}\right) = \cos^2\left(\dfrac{3\pi}{4}\right)$

18. $\tan^2\left(\dfrac{3\pi}{4}\right) = \sec^2\left(\dfrac{3\pi}{4}\right) - 1$

Use the definitions of the trigonometric functions to prove each identity.

19. $\sec\theta = \dfrac{1}{\cos\theta}, \quad \cos\theta \neq 0$

20. $\cot\theta = \dfrac{1}{\tan\theta}, \quad \tan\theta \neq 0$

21. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}, \quad \sin\theta \neq 0$

22. $1 + \cot^2\theta = \csc^2\theta$

23. $1 + \tan^2\theta = \sec^2\theta$

24. $\cos(-\theta) = \cos\theta$

25. $\sin\theta\csc\theta = 1$

26. $\sin\theta = \cos\theta\tan\theta$

27. $\cos^2\theta = 1 - \sin^2\theta$

28. $\sin^2\theta = 1 - \cos^2\theta$

Verify each identity for the given angle measure.

29. $\sin 45° \cot 45° = \cos 45°$

30. $\tan 60° = \sin 60° \sec 60°$

31. $\sin 135° \cos 135° \tan 135° = 1 - \cos^2 135°$

32. $\cos 30° \cot 30° = \csc 30° - \sin 30°$

In triangle *ABC*, sides *a*, *b*, and *c* measure 9, 40, and 41, respectively.

33. Show that triangle *ABC* is a right triangle.

34. Verify the identity $\cos A = \dfrac{\sin A}{\tan A}$ for triangle *ABC*.

35. Verify the identity $\csc B \cos B \tan B = 1$ for triangle *ABC*.

36. Verify the identity $\sec B \sin B \cot B = 1$ for triangle *ABC*.

Applications

37. **Engineering** A 20.2-m walkway between two buildings is higher at one end than at the other. The sine of the angle of elevation is 0.1386. Use a Pythagorean identity to find the cosine of that angle. Then find the distance between the buildings.